

Scalable Probabilistic Deterioration Model based on Visual Inspections and Structural Attributes from Large Networks of Bridges

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Abstract

Visual inspections of large networks of bridges yield millions of data points scattered across thousands of structural elements. Alongside visual inspections, structural attributes such as age, location and traffic load provide contextual information about the deterioration patterns in the network. Leveraging this network-scale data for modeling deterioration is challenging, especially when each structural element has few inspections over a long period of time. Moreover, as new bridge information and inspections are added each year, it is strictly important for deterioration models to be scalable. This paper addresses these challenges by proposing a scalable probabilistic approach for modeling deterioration of large networks of bridges. The new framework consists of state-space models (SSM) for modeling the deterioration based on visual inspections and a Bayesian neural network (BNN) that factors-in information about structural attributes. The role of the BNN model is to learn the mapping between the initial distribution of the deterioration speed and the structural attributes of each bridge. The new framework is shown to be computationally efficient and can seamlessly incorporate a large number of structural attributes, which alleviates the need for feature selection. In addition, the proposed framework incorporates a new approach for learning the inspectors' uncertainty parameters which is shown to provide better generalization. The experiments in this study are based on real data from the network of bridges in the province of Quebec, Canada.

1 Introduction

Deterioration of transportation infrastructure due to aging, usage, and environmental exposure is an issue faced by most industrialized countries [4]. Addressing this issue can be done by making timely maintenance decisions, which primarily rely on information from structural health monitoring systems [16]. The predominant approach to monitoring the health of bridges is based on periodic visual inspections that assess their structural health at the structural element level [1, 20, 3]. Even though inspections of bridge networks produce millions of inspection data points, they are spread over thousands of structural components. Effectively, this implies that many structural elements have few inspections over long periods of time.

The scarcity of inspection data for each structural element is further compounded by the subjective nature of inspections, which results in highly variable observations [11, 1, 24, 23]. For instance, two inspectors may assign different condition ratings to the same structural element depending on their level of experience [21]. Despite these challenges, visual inspection-based deterioration models remain to be an essential component in decision-making and maintenance planning [22, 15]. Notable examples include discrete Markov models [25, 2, 33, 35], approaches based on variational inference [5], regression methods [18, 29, 31], and state-space models (SSM) [11]. Among the existing frameworks, the SSM model stands out as it provides modelling advantages such as, quantifying the uncertainty associated with each inspector and the probabilistic estimation of the deterioration speed [21, 13]. Nonetheless, the SSM model long-term predictive capacity can be limited due to relying only on visual inspection data [11]. Other approaches in the literature ventured to reduce the reliance on visual inspections by

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integrating physical models like an analytical carbonation-induced corrosion model [34] or an empirical chloride-induced corrosion model [8]; however, they require extensive knowledge about each structure (e.g., information from original design documents, various tests and measurements, etc.), thus limiting their scalability.

Other extensions of deterioration models includes incorporating contextual information such as material, age and location alongside visual inspection data [18, 29, 31, 27]. Information about the aforementioned structural attributes can be used in predicting or refining the estimates of the health condition [18, 12]. Existing examples include the coupling of SSM framework [11] with kernel regression (KR) to improve the estimated deterioration speed and refine the deterioration model forecast. While this hybrid model maintains the advantages attained from using the SSM framework, it suffers from computational inefficiency and limited capacity in incorporating structural attributes [12]. Primarily, the SSM-KR framework only allows for up to six structural attributes and takes significant computing resources and time to train [12, 14].

This paper proposes a hybrid deterioration model that couples the SSM with a Bayesian neural network (BNN) to effectively incorporate the structural attributes in the deterioration analyses. This new hybrid model provides direct solutions to the highlighted limitations in the SSM-KR framework by, 1) being computationally efficient and significantly reduce the compute time for training, and 2) being scalable which enables considering all available information about the structural attributes. Furthermore, the parameters of the proposed deterioration model are estimated using a new estimation framework, which reduces the total number of model parameters and is shown to improve the model predictive capacity when evaluated on an independent test set. The numerical analyses in this study includes comparing the performance of the SSM-BNN model against the existing SSM-KR model. The validation analyses are performed using data from the network of bridges in the Quebec province, Canada.

2 Problem Context

A network consists of a set of bridges $\{\mathcal{B}_1, \mathcal{B}_2, \dots, \mathcal{B}_B\}$ with each bridge \mathcal{B}_j encompassing multiple structural elements e_p^j . Each structural elements can be characterized by its health state and its structural attributes. The health state in this context is evaluated over time using visual inspections, while the structural attributes encompass attributes directly related to the element (e.g., material) and attributes that are related to the bridge (e.g., bridge location). Figure 1 provides an illustrative example about the bridge data considered in this study.

Figure 1, shows that despite variations in the health condition among the structural elements, the deterioration speed patterns can be similar between structures sharing similar structural attributes. The main objective in this study is to effectively learn the relationship between structural attributes and the deterioration speed across multiple bridges, in order to improve the predictive capacity of deterioration models based on visual inspections.

3 Background

This section provides the theoretical background for the existing methods related to the proposed probabilistic deterioration model.

3.1 State-space Models

State-space models (SSM) describe the behavior of a system over time using probabilistic transition and observation models [9]. They have been shown to be effective at modeling infrastructure deterioration on a network-scale [11], where the transition model describes the physical deterioration process following,

$$\underbrace{\mathbf{x}_t = \mathbf{A}\mathbf{x}_{t-1} + \mathbf{w}_t}_{\text{transition model}}, \quad \underbrace{\mathbf{w}_t : \mathbf{W} \sim \mathcal{N}(\mathbf{w}; \mathbf{0}, \mathbf{Q})}_{\text{process errors}}. \quad (1)$$

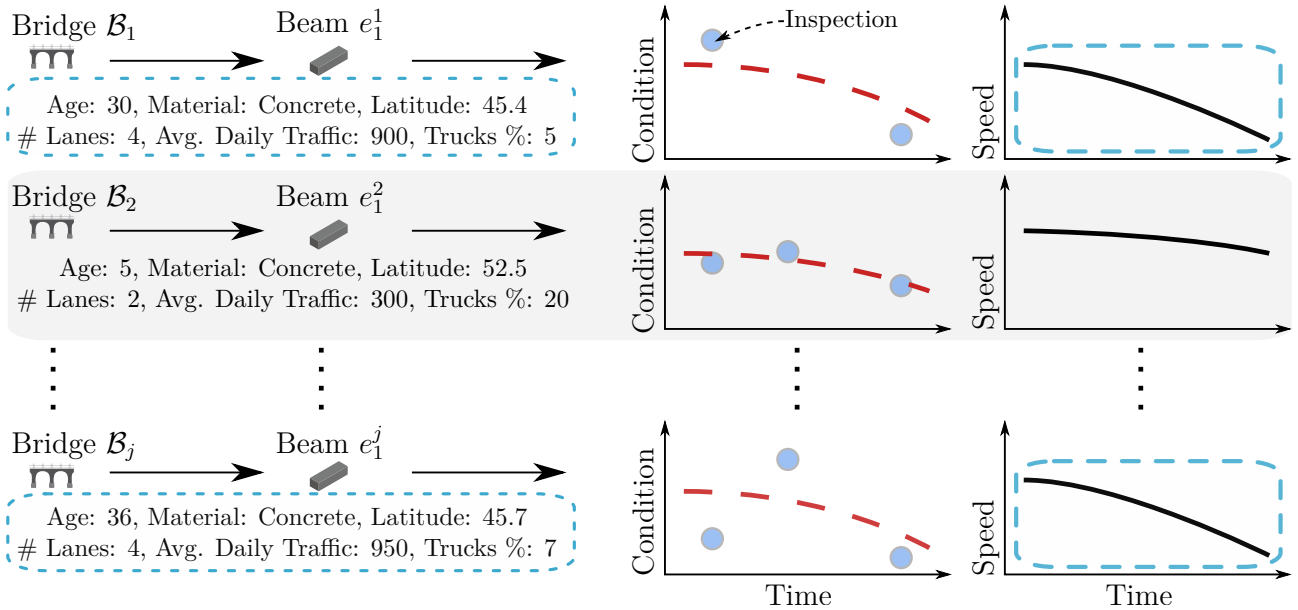


Figure 1: Illustrative example for bridge data which shows structural attributes associated with each structural element (left) alongside the visual inspection data (right, blue points). The example demonstrates that while the deterioration condition may vary among structural elements, the patterns of deterioration speed can be similar, especially for structural elements sharing similar structural attributes. Structural elements with similarities are highlighted with a dashed-line rectangle.

Here, $\mathbf{x}_t = [x_t \dot{x}_t \ddot{x}_t]^\top$ is the hidden state vector at time t containing the condition x_t , speed \dot{x}_t , and acceleration \ddot{x}_t of a structural element, \mathbf{A} is the state transition matrix, \mathbf{w}_t is the process error, and \mathbf{Q} is the process error covariance matrix. The state transition matrix and its associated process error covariance matrix are derived assuming a constant-acceleration kinematic model,

$$\mathbf{A} = \begin{bmatrix} 1 & \Delta t & \frac{\Delta t^2}{2} \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{Q} = \sigma_W^2 \begin{bmatrix} \frac{\Delta t^4}{4} & \frac{\Delta t^3}{2} & \frac{\Delta t^2}{2} \\ \frac{\Delta t^3}{2} & \Delta t^2 & \Delta t \\ \frac{\Delta t^2}{2} & \Delta t & 1 \end{bmatrix}, \quad (2)$$

where Δt is the time step between successive states and σ_W^2 is the variance of the process error [11]. The uncertainty of visual inspections are characterized by the observation model described by,

$$\underbrace{y_t = \mathbf{C}\mathbf{x}_t + v_t}_{\text{observation model}}, \quad \underbrace{v_t : V \sim \mathcal{N}(v; \mu_V(i), \sigma_V^2(i))}_{\text{observation errors}}, \quad (3)$$

where y_t represents the observation at time t , $\mathbf{C} = [1 \ 0 \ 0]$ is the observation matrix, and v_t is the observation error with $\mu_V(i)$ and $\sigma_V^2(i)$ representing the relative bias and variance of the i -th inspector [21]. The deterioration state at each time step t is inferred using the Kalman filter (KF) [19] and Rauch-Tung-Striebel (RTS) smoother [28], which are detailed in B. Using the KF to model deterioration requires imposing two constraints on the state estimates to ensure the condition x_t is monotonically decreasing over time and to keep the model predictions within a predefined range of values [11]. The first constraint is enforced at each time step by restricting the deterioration rate \dot{x}_t to be negative using a probability density function (PDF) truncation method [30]. The second constraint is imposed using space transformation which preserves the hidden states and observations within a predefined range of health conditions $[l, u]$. Further details about the space transformation are provided in C. Although the SSM framework can effectively model deterioration, it solely relies on the visual inspection data without incorporating information about structural attributes [11].

3.2 State-Space Models with Kernel Regression

Augmenting the SSM framework with a kernel-based regression (KR) model enables taking into account information about structural attributes. The purpose of KR is to include structural attributes \mathbf{z} in

the deterioration analyses to improve the prior for each element's initial state $\mathbf{x}_0 = [x_0 \ \dot{x}_0 \ \ddot{x}_0]^\top$ by sharing information between structures. More precisely, KR predicts the deterioration speed \dot{x}_0 based on each element's attributes that are defined for each bridge by a vector $\mathbf{z}_j = [z_j^1 \ \dots \ z_j^Q]^\top$. The KR method involves discretizing each of the Q covariates' domains with M reference points, which are then permuted to form a grid of $N \equiv M^Q$ points denoted by $\mathbf{G} = [\mathbf{g}_1 \ \dots \ \mathbf{g}_N]^\top \in \mathbb{R}^{N \times Q}$. Each i -th grid point in \mathbf{G} corresponds to a unique combination of the reference points and is associated with a deterioration rate $\dot{x}_i \in \dot{\mathbf{x}}_{\mathbf{z}} = [\dot{x}_1 \ \dots \ \dot{x}_N]^\top$. KR estimates each element's deterioration rate \dot{x}_0 based on the proximity of its attributes \mathbf{z}_j to different grid points using a weighted sum given by,

$$\dot{x}_0 = \mathbf{a}^\top \dot{\mathbf{x}}_{\mathbf{z}} + w_0, \quad w_0 : W_0 \sim \mathcal{N}(0, \sigma_{W_0}^2), \quad (4)$$

where w_0 is the process error and $\mathbf{a} = [a_1 \ a_2 \ \dots \ a_N]^\top$ is the vector of weights, which is obtained using a multivariate kernel function $\mathbf{k}(\cdot)$ following

$$a_i = \frac{\mathbf{k}(\mathbf{z}_j, \mathbf{G}_{(i)}, \boldsymbol{\ell})}{\sum_{n=1}^N \mathbf{k}(\mathbf{z}_j, \mathbf{G}_{(n)}, \boldsymbol{\ell})}, \quad i = 1, \dots, N, \quad (5)$$

where $\boldsymbol{\ell} = [\ell_1 \ \dots \ \ell_Q]^\top$ are the kernel bandwidths and $\mathbf{G}_{(i)}$ is the i -th point on the grid. The kernel function $\mathbf{k} : \mathbb{R}^Q \rightarrow \mathbb{R}$ is defined as a product of univariate kernels $k(\cdot)$ given by

$$\mathbf{k}(\mathbf{z}_j, \mathbf{G}_{(i)}, \boldsymbol{\ell}) = k\left(\frac{z_j^1 - g_i^1}{\ell_1}\right) \cdot \dots \cdot k\left(\frac{z_j^Q - g_i^Q}{\ell_Q}\right). \quad (6)$$

The parameter estimation of the SSM-KR model is detailed in the work of Hamida and Goulet [12]. Although the use of KR can improve the predictive capacity of the deterioration state, the SSM-KR model has limitations related to scalability and computationally efficiency. Namely, the KR model can only incorporate a limited number of structural attributes as its covariates, and it requires significant computational resources and training time. Additionally, the performance of KR is highly dependent on the selection of its hyper-parameters, such as kernel types, which can be a lengthy process to identify [12]. Overcoming the inherent limitations in the SSM-KR model is only possible by replacing KR with a probabilistic regression approach that is compatible with SSM, scalable and computationally efficient. Bayesian neural networks (BNN) offer such an alternative, as they provide a flexible framework for probabilistic regression that aligns well with the aforementioned requirements.

4 Methodology

This section presents a new hybrid framework for modeling infrastructure deterioration based on state-space models and a Bayesian neural network (BNN) trained via tractable approximate Gaussian inference (TAGI).

4.1 Coupling Bayesian Neural Networks with State-space Models

In the proposed SSM-BNN model, the BNN component is used to enhance the SSM predictions by learning the relation between the attributes of structural elements (age, location, etc.) and their deterioration speed. This learned relation is used to define a better prior distribution for the initial deterioration speed of each structural element. To illustrate this, Figure 2 shows an example of a fully-trained SSM-BNN model providing the estimates for the deterioration state of a structural element e_p^j in the bridge \mathcal{B}_j . The estimation of the deterioration state starts with transforming the visual inspection data $\tilde{\mathbf{y}}_{t,p}^j = [\tilde{y}_{1,p}^j \ \dots \ \tilde{y}_{T,p}^j]^\top \in [l, u]$ into an unbounded domain $\mathbf{y}_{t,p}^j = [y_{1,p}^j \ \dots \ y_{T,p}^j]^\top \in (-\infty, \infty)$ using the space transformation function $o(\cdot)$ detailed in C. Here, l represents the worst damage condition and u represents the perfect condition. The transformed observations $\mathbf{y}_{t,p}^j$ are then passed to the SSM, which relies on the Kalman filter (KF) to estimate the element's deterioration state over time. The KF starts with a guess about the element's initial state and propagates this prior knowledge forward in time using the transition model, generating predictions that are updated using the observations. Defining the initial state of an element $\mathbf{x}_{0,p}^j = [x_{0,p}^j \ \dot{x}_{0,p}^j \ \ddot{x}_{0,p}^j]^\top$ is an important aspect that affects the

quality of the KF predictions, since there are only few observations (or inspections) per element. While it is possible to derive an estimate for the initial condition using the observation data, the deterioration speed is not observable, hence it is challenging to define an initial deterioration speed for each structural element [11]. The BNN model provides an estimate for the initial deterioration speed $\dot{x}_{0,p}^j \sim \mathcal{N}(\mu_z, \sigma_z)$ by recursively learning a mapping between the inferred deterioration speed values and the structural attributes \mathbf{z}_p^j of each structural element. Further details about the recursive learning procedure are provided in the next section.

After defining the element's initial state $\mathbf{x}_{0,p}^j$, it is passed into the KF, together with the element's transformed observations $\mathbf{y}_{t,p}^j$. The KF then predicts the deterioration state of the element over time $\{\mathbf{x}_{0,p}^j, \dots, \mathbf{x}_{T,p}^j\}$, where T denotes the time-horizon of the prediction. At each time step of the KF, the deterioration speed $\dot{x}_{t,p}^j$ is ensured to be negative by requiring that $\mu_{t,p}^j + 2\sigma_{t,p}^j \leq 0$ [11]. If this condition is not satisfied, the deterioration speed constrained to the negative domain by truncating its PDF [11, 30]. Following the completion of the KF, the RTS smoother is used to refine the KF state estimates. The smoothed state estimates are then back-transformed to the original space $\{\tilde{\mathbf{x}}_{0,p}^j, \dots, \tilde{\mathbf{x}}_{T,p}^j\}$ using the inverse of the transformation function $o^{-1}(\cdot)$ (see C).

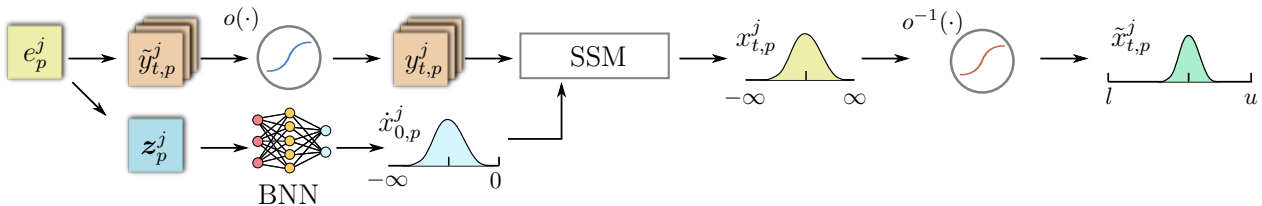


Figure 2: Overview of the procedure for estimating the deterioration state of the p -th element from the j -th bridge, denoted by e_p^j , using the proposed SSM-BNN framework. The original visual inspections of the element $\tilde{y}_{t,p}^j$ are first transformed into an unbounded domain using the sigmoid function $o(\cdot)$, yielding the transformed observations $y_{t,p}^j$. The BNN then uses the structural attributes of the element \mathbf{z}_p^j to define its initial deterioration rate $\dot{x}_{0,p}^j$. The initial state $\mathbf{x}_{0,p}^j$ and the transformed observations $y_{t,p}^j$ are then passed into SSM to predict the element's deterioration state over time $\mathbf{x}_{t,p}^j = [x_{t,p}^j, \dot{x}_{t,p}^j, \ddot{x}_{t,p}^j]^\top$ for $t \in \{1, 2, \dots, T\}$, where T denotes the time-horizon of the prediction. These predicted states are finally back-transformed to the original space using $o^{-1}(\cdot)$, resulting in $\tilde{\mathbf{x}}_{t,p}^j = [\tilde{x}_{t,p}^j, \tilde{\dot{x}}_{t,p}^j, \tilde{\ddot{x}}_{t,p}^j]^\top$.

4.2 Recursive Estimation of the BNN Parameters

The parameters of the BNN are learned using a recursive procedure due to the fact that the response variable represented by the deterioration speed is not observed but rather is inferred. Figure 3 illustrates the full recursive estimation procedure. Initially, at iteration $i = 0$, the state vector for each structural element is defined by $\mathbf{x}_{0,p}^j \sim \mathcal{N}(\boldsymbol{\mu}_{0,p}^j, \boldsymbol{\Sigma}_{0,p}^j)$, with the expected value relying mainly on the observation $y_{1,p}^j$ at time $t = 1$ such that, $\boldsymbol{\mu}_{0,p}^j = [y_{1,p}^j, 0, 0]^\top$ while the covariance is defined as $\boldsymbol{\Sigma}_{0,p}^j = \text{diag}((\sigma_{0,p}^j)^2, (\dot{\sigma}_{0,p}^j)^2, (\ddot{\sigma}_0)^2)$. All the components in the covariance matrix correspond to model parameters and their estimation is detailed in the next section. The initial state vector $\mathbf{x}_{0,p}^j$ of each element is then passed into the SSM framework composed of the Kalman filter and smoother. The SSM returns the smoothed estimates of the initial deterioration speed $f(\dot{x}_{0,p}^j | y_{1:T,p}^j) = \mathcal{N}(\mu_{0|T,p}^j, (\sigma_{0|T,p}^j)^2)$ that correspond to each structural element e_p^j . The smoothed initial speed estimates are thereafter used as target variable in training a BNN model with the structural attributes \mathbf{z}_p^j as input covariates. Training the BNN model is done using tractable approximate Gaussian inference (TAGI) which is a sampling-free and gradient-free approach to learn the model BNN model parameters [10].

Following the BNN training step, the BNN model is used to provide only the expected value for the initial speed such that, $\boldsymbol{\mu}_{0,p}^j = [y_{1,p}^j, \mu_{z,p}^j, 0]^\top$, while the initial covariance matrix $\boldsymbol{\Sigma}_{0,p}^j$ remains the same. This is done to avoid having initial variances of the deterioration speed that are too small during the recursive learning procedure. The steps of obtaining the smoothed initial speed estimates $(\mu_{0|T,p}^j, \sigma_{0|T,p}^j)$

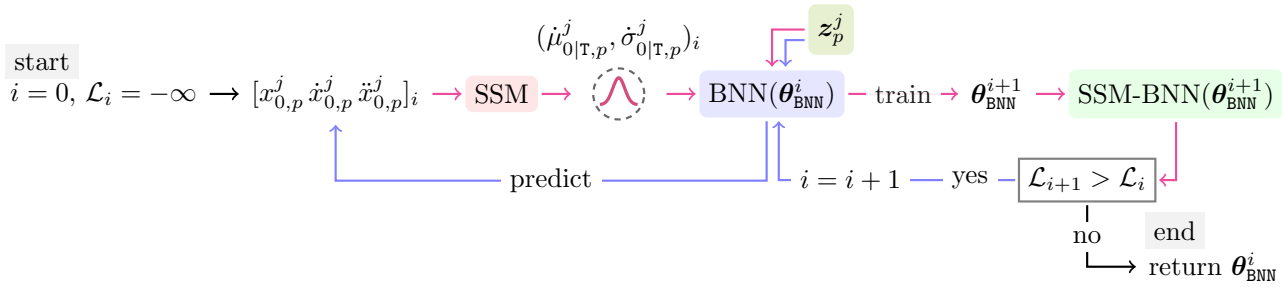


Figure 3: The recursive process of estimating the BNN parameters θ_{BNN} . At $i = 0$, the initial states $[x_{0,p}^j, \dot{x}_{0,p}^j, \ddot{x}_{0,p}^j]_i$ are passed to the SSM to obtain the smoothed initial speeds $(\mu_{0|T,p}^j, \dot{\sigma}_{0|T,p}^j)_i$. These are then used to train the BNN, so that it learns to predict them based on the structural attributes z_p^j . The SSM-BNN model then evaluates the log-likelihood of the validation set \mathcal{L}_{i+1} and compares it to the previous iteration \mathcal{L}_i . If there is an improvement, the BNN uses the structural attributes z_p^j to define the priors for the initial deterioration speeds $\dot{x}_{0,p}^j$, and the recursive loop is continues until no further improvements in \mathcal{L}_i .

and re-training the BNN model are repeated until there is no improvement in the log-likelihood \mathcal{L} of the validation set.

4.3 Deterioration Model Parameters & Hyper-parameters

The parameters of the SSM-BNN model can be grouped in the set θ which includes, $\theta = \{\theta_V, \sigma_W, \theta_{\text{BNN}}, \theta_0\}$. The set θ_V corresponds to the observation error parameters, where each inspector has a unique error term defined by the expected value μ_{V_i} and standard deviation σ_{V_i} [21]. The parameter σ_W represents the standard deviation of the process noise, and the parameters set θ_{BNN} corresponds to the weights and biases of the BNN model. The set θ^0 contains the parameters associated with the initial covariance matrix $\Sigma_{0,p}^j$, which are adopted from a previous work by [11]. The parameters in θ^0 include the variance of the initial condition σ_0^2 , initial acceleration $\ddot{\sigma}_0^2$, and $\{p_1, p_2\}$ which define a linear relation between the variance of the deterioration speed and the deterioration condition as in, $\dot{\sigma}_0^2 = p_1^2 (u - \tilde{\mu}_1) + p_2^2$. The intuition of the linear model is to counteract the low number of observations in the time series by incorporating an informative prior, so that the elements with a near perfect condition ($u - \tilde{\mu}_1 \approx 0$) will have a lower uncertainty about their deterioration speed compared to worn out elements, where $\tilde{\mu}_1$ corresponds to the expected value of the condition at time $t = 1$ [11].

The hyper-parameters of the SSM-BNN include the space transformation parameter n , which governs the shape of the transformation function $o(\cdot)$ used to bound the hidden states and observations [11], and the γ scaling factor which affects the variance of the deterioration speed, and is only applied at the first iteration of the recursive estimation procedure described earlier. The scaling factor can be determined empirically from the range [1, 10], and is multiplied by the parameter value p_1 in order to provide a broader prior for $\dot{x}_{0,p}^j$, enabling the SSM model to have higher flexibility in updating $\dot{x}_{t,p}^j$ during the initial stage of recursive estimation.

4.4 Estimation of the SSM-BNN Parameters

The computational efficiency of the BNN model enables formulating a new parameters estimation scheme that offers better generalizability. Unlike the existing parameter estimation framework which only allows learning the model parameters using data from one structural category at a time (e.g., beams only), the new parameters estimation framework enables learning model parameters while taking into account information from all structural categories at once. This is especially important when estimating the inspectors parameters, where in a single structural category (e.g., bridge deck) an inspector may have few data points compared to other structural categories (e.g., beams).

Accordingly, the proposed parameter estimation framework updates the inspectors parameters θ^V based on data from all structural categories, while the remaining model parameters are specific to each structural category. The estimation procedure is divided into three stages: 1) estimation of the

inspectors' parameters $\theta_V = \{\mu_{V_{1:i}}, \sigma_{V_{1:i}}\}$, 2) estimation of the process noise σ_W along with the initial covariance parameters $\theta_0 = \{\sigma_0, \ddot{\sigma}_0, p_1, p_2\}$, and 3) estimation of the BNN model parameters θ_{BNN} . In each of the aforementioned stages, a *maximum log-likelihood estimate (MLE)* approach is utilized in the parameter estimation [11], with the log-likelihood function defined as,

$$\mathcal{L}(\theta) = \sum_{j=1}^B \sum_{p=1}^{E_j} \sum_{t=1}^{T_p} \ln f(y_{t,p}^j | y_{1:t-1,p}^j, \theta), \quad (7)$$

where B denotes the total number of bridges in the network, E_j denotes the number of structural elements in the j -th bridge, and T_p denotes the number of inspections for the p -th element in the j -th bridge. The full parameter estimation procedure is summarized in Algorithm 1.

Algorithm 1: A pseudocode for the new parameter estimation framework.

```

for  $n \in \{1, 2, 3, 4, 5\}$  do
  for  $\mathcal{C}_m \in \{\text{beams, slabs, columns, ...}\}$  do
    optimize  $\theta_0^{(m)}, \sigma_W^{(m)}, \sigma_V^{(m)}$  using NR;
  end
  while  $\Delta \sum_m \mathcal{L}^{(m)} \geq 0.1\%$  do
    optimize  $\theta_V = \{\mu_{V_{1:i}}, \sigma_{V_{1:i}}\}$  using approximate Gaussian inference;
    for  $\mathcal{C}_m \in \{\text{beams, slabs, columns, ...}\}$  do
      while  $\Delta \mathcal{L}^{(m)} \geq 0.1\%$  do
        optimize  $\theta_0^{(m)}, \sigma_W^{(m)}$  using NR;
        optimize  $\theta_{\text{BNN}}^{(m)}$  using recursive estimation
      end
    end
  end
end

```

The parameter estimation starts by assigning a value to the hyper-parameter n which is an integer defined in the range $n \in \{1, 2, 3, 4, 5\}$ [11]. For each structural category $\mathcal{C}_m \in \{\text{beams, slabs, columns, ...}\}$, a preliminary parameter estimation step is performed to learn the parameters $\sigma_W^{(m)}, \theta_0^{(m)}$ and $\sigma_V^{(m)}$ which represents to standard deviation of the observation error. Initially, all inspectors are assumed to have the same observation error defined by $V_i \sim \mathcal{N}(0, (\sigma_V^c)^2)$ [11]. Estimating the aforementioned parameters is done using the Newton-Raphson (NR) gradient optimization algorithm while relying on Equation 7.

Following this preliminary step, the observation error is further refined by quantifying the observation error associated with each inspector defined in the set of parameters θ_I . The estimation of θ_V is performed using the approximate Gaussian variance inference (AGVI), which is adopted from the work of Blanche et al. [21]. The use of AGVI in this framework remains fundamentally the same, with the sole distinction being the inclusion of inspector's inspection data from all structural categories, as opposed to a singular structural category. This modification allows for a unified set of inspectors' uncertainty parameters across all structural categories; which is different from previous applications where each structural category had a distinct set of inspector uncertainty parameters [14, 21]. Further details about learning the inspectors uncertainties and the AGVI approach are provided in D.

Following the estimation of θ_V , the BNN model parameters θ_{BNN} are estimated using the recursive estimation approach described earlier. The sequential estimation of $\theta_V, \theta_0, \sigma_W^{(m)}$, and θ_{BNN} is repeated until the improvement in the overall validation-set log-likelihood is negligible (e.g., $\leq 0.1\%$). It should be noted that the validation set is an independent set, where the structural elements of the same structure are not simultaneously present in the training set and the test set.

5 Case studies

This section presents case studies highlighting the effectiveness of the parameter estimation and the performance of the proposed SSM-BNN model. The case studies rely on real data from the network of bridges in Quebec, Canada, as well as synthetic data generated for verification purposes.

5.1 Data Description

5.1.1 Real Data:

In this paper, infrastructure deterioration is modeled based on visual inspections and structural attributes from a network of approximately 10000 bridges located in the province of Quebec, Canada. The hierarchy of the data is illustrated in Figure 4, where each bridge \mathcal{B}_j is made up of different structural categories $\{\mathcal{C}_1^j, \dots, \mathcal{C}_m^j, \dots, \mathcal{C}_M^j\}$, and each structural category is composed of multiple structural elements, $\mathcal{C}_m^j = \{e_1^j, \dots, e_p^j\}$. The visual inspection data denoted by $\tilde{y}_{t,p}^j$ correspond to observations of the elements's health condition over time t . In this context, \tilde{y} is defined on a continuous domain, where $\tilde{y} = 100$ represents a perfect condition and $\tilde{y} = 25$ signifies the poorest condition. The inspection frequency differs from one bridge to another, with ranges from once a year to once every five years [26]. In total, the majority of the bridges have four or five inspections performed in the period from 2007 to 2023.

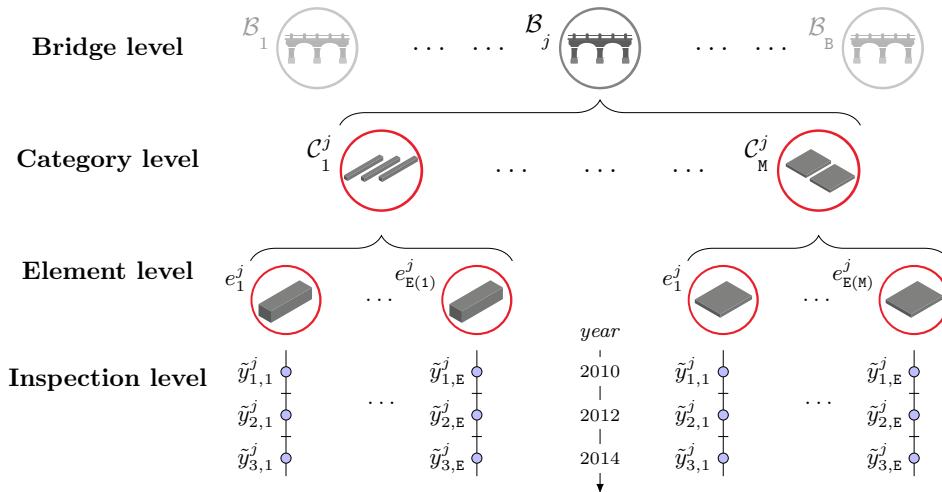


Figure 4: Graphical illustration of the components making up the bridge database, with each bridge \mathcal{B}_j encompassing multiple structural categories \mathcal{C}_m^j and each category containing multiple structural elements. The visual inspections are performed on an element level, whereby an element e_p^j is assigned a condition rating of $\tilde{y}_{t,p}^j$ at the time step t .

The structural attributes associated with each structural element e_p^j are denoted by z_p^j and correspond to the material of the structural element, age, latitude, longitude, total length of the bridge, slab length, total width of the bridge, surface area, number of lanes, percentage of trucks, annual average of daily traffic (AADT) and average health condition. A summary for the structural attributes considered in this study is provided in Table 1.

5.1.2 Synthetic Data:

The synthetic data utilized in this study has comparable quantitative and qualitative characteristics to the real visual inspection data. The quantitative similarities include similarities in the number of synthetic structural elements, the number of inspections per structural element, and the total number of unique observation errors (i.e., number of inspectors). The qualitative similarities on the other hand equates to similarity in the deterioration curve over time and the average life span of the structural element [11].

Table 1: Summary of the structural attributes associated with each structural element.

covariates	value ranges	unit
$z_{p,1}^j$: material	[0, 1]	-
$z_{p,2}^j$: age	[0, 165]	years
$z_{p,3}^j$: latitude	[44.99, 58.67]	-
$z_{p,4}^j$: average health condition	[28, 100]	-
$z_{p,5}^j$: longitude	[-79.51, -57.25]	-
$z_{p,6}^j$: total length	[3.674, 1801.4]	meters
$z_{p,7}^j$: slab length	[3.674, 1801.4]	meters
$z_{p,8}^j$: total width	[3.35, 120]	meters
$z_{p,9}^j$: surface area	[17, 52420]	square meters
$z_{p,10}^j$: number of lanes	[0, 13]	lanes
$z_{p,11}^j$: percentage of trucks	[0, 100]	-
$z_{p,12}^j$: annual average daily traffic	[0, 178000]	vehicle per day

In this paper, a total of $E = 20000$ synthetic beam elements are considered, with each element having 4 to 5 inspections over a time span of 10 years. The elements deterioration curves are designed to have an average service life of 60 years. The total number of synthetic inspectors is $I = 300$, with each inspector’s error defined by $v_i : V_i \sim \mathcal{N}(\mu_{V_i}, \sigma_{V_i}^2)$. The true deterioration curves for each synthetic element are generated using the transition model defined in Equation 1 with the standard deviation of the process noise set to $\sigma_W = 5 \times 10^{-3}$. While the synthetic inspection data is generated using the observation model defined in Equation 3 by sampling synthetic observations from the true synthetic deterioration states. The parameters associated with the observation errors of inspectors are sampled from uniform distributions, following $\mu_{V_i} \sim \mathcal{U}(-4, 4)$ and $\sigma_{V_i} \sim \mathcal{U}(1, 6)$. To ensure that the deterioration curves exhibit the same qualitative characteristics of the real deterioration, several criteria are enforced on the synthetic elements’ states, which are adopted from the work of Hamida and Goulet [11]. Alongside the synthetic visual inspection data, a synthetic structural attribute z^j is generated using: $z^j = \ln(|\dot{x}_0^j|) + w_0$, where w_0 is an error term described by $w_0 : W_0 \sim \mathcal{N}(0, 0.1^2)$, [12]. This relation results in unevenly distributed synthetic attribute z^j with a long-tail distribution (see Figure 5), allowing to effectively verify the robustness of the proposed framework.

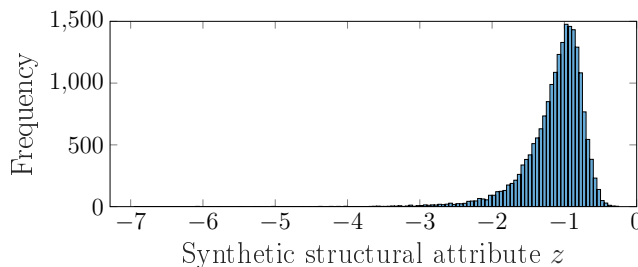


Figure 5: Histogram of the synthetic structural attribute generated using $z^j = \ln(|\dot{x}_0^j|) + w_0$, $w_0 : W_0 \sim \mathcal{N}(0, 0.1^2)$ adopted from the work of Hamida and Goulet [12].

5.2 Verification of the SSM-BNN Model Using Synthetic Data

The SSM-BNN framework is verified on synthetic data using a neural network architecture and configuration defined in A. In this example, the BNN model is trained with a single input variable corresponding to the synthetic structural attribute z^j , and a single output variable which is the deterioration speed. The training data is split into training and validation sets using an 85/15 split to avoid overfitting, and is standardized to have a zero mean and unit variance.

The parameters of the SSM-BNN model are estimated using the steps described in the pseudocode in Algorithm 1. While all the steps in the pseudocode are equally important, this section will focus mainly on the capacity of the BNN model in learning the relationship between the synthetic structural attribute z^j and the deterioration speed. Figure 6 illustrates the latent relation between the synthetic structural attribute z^j and the initial deterioration speed \dot{x}_0 as learned by the BNN model, along with the training data and the ground truth.

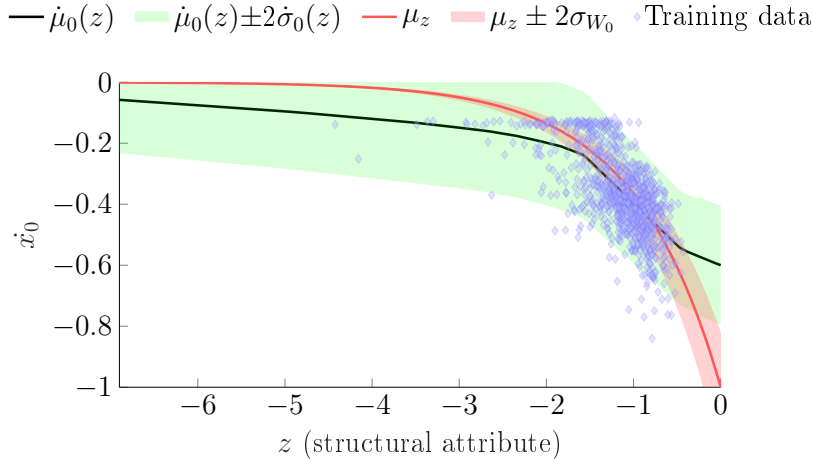


Figure 6: The latent relation between the synthetic structural attribute z and the deterioration speed \dot{x}_0 learned by the BNN. The black line represents the expected values predicted by the BNN with the shaded green area representing uncertainty. The true relation is depicted by the red line with the shaded red area representing its variability. Having this variability makes the synthetic attributes resemble reality, where structures with different attributes can deteriorate at the same speed.

From Figure 6, the training data represented by the blue diamond-shapes corresponds to the smoothed estimates of the initial deterioration speed $\dot{x}_{0|T}$ for each synthetic structural elements. On the other hand, the red curve represents the true relation between $\dot{x}_{0|T}$ and z^j . The BNN model inference for the aforementioned relation is depicted by the black line with the green confidence interval. The BNN model inference is achieved by relying on the recursive estimation procedure described in Section 4.2. From Figure 6, the true speed lies within two standard deviations of the mean predicted by the BNN for nearly the full range of inputs, in spite of the input z^j having a range of values $z^j \in [-7, -2]$ with a lower density of training data.

Following the estimation of the model parameters, the performance of the SSM-BNN model is compared against the existing SSM-KR model, which is trained and configured based on the work of Hamida and Goulet [12]. The performance comparison focuses on the predictive capacity of each model in estimating the initial deterioration speed as well as the total time required for estimating the model parameters. Figure 7 shows a scatter plot for the model predictions versus the true initial speed values which is generated based on a random sample of 500 synthetic structural elements.

From Figure 7 it is shown that the estimates of the SSM-BNN model have a better alignment with the true deterioration speed values, especially for lower deterioration speeds (i.e., $\dot{x}_0 < -0.3$). This is demonstrated by the equal spread of the SSM-BNN predictions around the diagonal line. As for the total training time, the SSM-BNN model required around 29 minutes to learn the full set of parameters, while the SSM-KR needed approximately 57 minutes.

5.3 Validation of the SSM-BNN Using Real Data

The proposed SSM-BNN model is initially trained using the visual inspection data and the structural attributes $z_{p,1:12}^j$ of beam structural elements from $B = 5998$ bridges. The visual inspections in the aforementioned dataset are performed by $I = 311$ inspectors. The beams's dataset encompass a training set with $E_{tr} = 48517$ elements, a validation set with $E_{val} = 2178$ elements, and a test set with $E_{ts} = 1089$. The architecture and configuration of the BNN model are described in A.

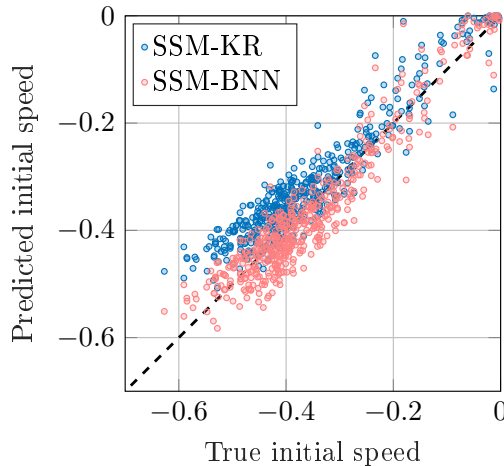


Figure 7: Scatter plot and histogram comparing the true initial deterioration speeds \dot{x}_0 of the synthetic elements with those predicted by SSM-BNN and SSM-KR.

Similar to the synthetic case, the input covariates and the response variable are standardized to have zero mean and unit variance. The output of the BNN model is a single variable representing the deterioration speed $\dot{x}_{0,p}^j$ at time $t = 0$, while the total number of the input covariates is $Z = 18$. The input covariates correspond to a one-hot encoded material covariate in addition to 11 structural attributes $z_{p,2:12}^j$ defined in Table 1. The material in the case of beam elements includes: weathering steel, regular steel, aluminum, wood, high-performance concrete, prestressed concrete, and regular concrete. An example that demonstrates the SSM-BNN model performance on a regular steel beam element is shown in Figure 8, where the model maintains a downward trend despite the upward trend of the first three inspection points.

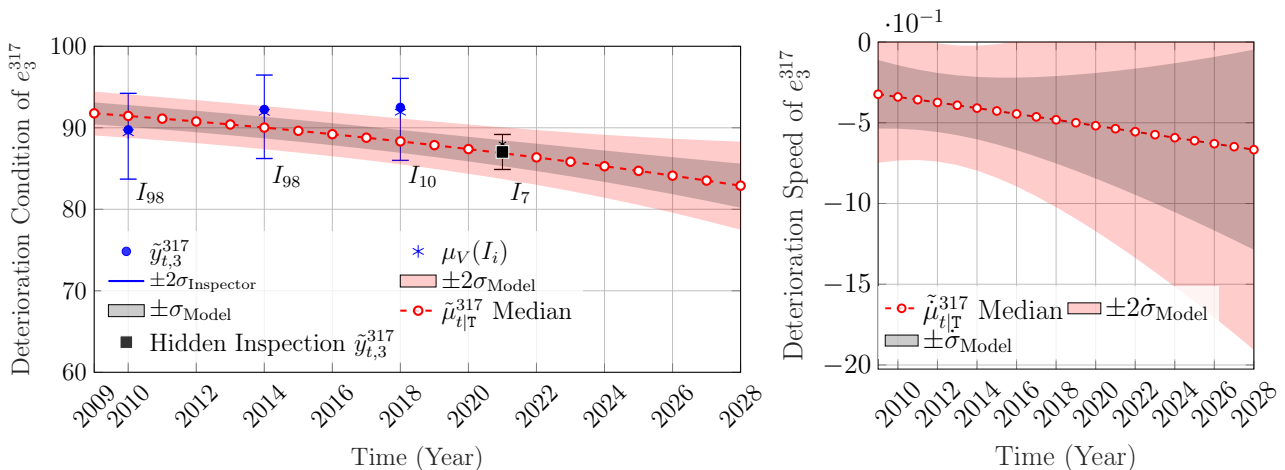


Figure 8: Estimates of deterioration state for condition (left) and deterioration speed (right) based on inspection data $\tilde{y}_{t,3}^{317} \in [25, 100]$ represented by the blue points. Error bars represent the inspectors' uncertainty estimates, while the black square correspond to a hidden inspection for testing.

From Figure 8, the inspection point represented by the black square is only used for validating the model predictive capacity and is not used in updating or estimating the model parameters. Figure 8 shows a good alignment between the model prediction and the hidden inspection point which equates to a good predictive capacity.

The analyses from the previous example can be expanded by evaluating the predictive capacity of the SSM-BNN model using hidden inspections data points from approximately 1000 beam elements taken from an independent test set of bridges. The performance metrics considered in this example are the log-likelihood and training time, which are reported in Table 2.

From Table 2, the SSM-BNN model is nearly two orders of magnitude faster while achieving

Table 2: The performance and training time of SSM-KR against SSM-BNN using only the beam elements.

model	log-likelihood	total training time (hrs)
SSM-KR	-14180	189
SSM-BNN	-13364	3

approximately 6% improvement in the log-likelihood. Note that the training times for the real and synthetic data are measured while using a system equipped with an Intel Xeon 6248R CPU, 256GB memory, and a NVIDIA Quadro RTX 5000 GPU.

5.4 Large-scale Application of the SSM-BNN Model

As mentioned in the methodology section, the SSM-BNN model benefits from a new parameter estimation procedure that allows having a unified set of inspector’s parameters θ_V across multiple structural categories. In this experiment, the scope of the analyses is limited to five structural categories: \mathcal{C}_1 : exterior walls, \mathcal{C}_2 : front walls, \mathcal{C}_3 : slabs, \mathcal{C}_4 : beams, and \mathcal{C}_5 : pavement. In total, there are $I = 325$ inspectors responsible for performing inspections across all bridges, with each structural category \mathcal{C}_m having approximately $I_m \approx 300$ inspectors. Accordingly, it is very likely for each inspector I_i to perform inspections on different structural categories. For example, inspector $I_{i=3}$ can be responsible for performing the inspections on beams, slabs and front walls in different years and on different bridges.

The existing parameter estimation procedure assumes that each structural category has a distinct set of parameters defined as in, $\theta_{\text{exist}} = \{\theta_V^{(1:5)}, \sigma_W^{(1:5)}, \theta_{\text{BNN}}^{(1:5)}, \theta_0^{(1:5)}\}$, where the superscript $(n:m)$ is a reference to the structural category index from \mathcal{C}_1 to \mathcal{C}_5 [14]. Inherently, this approach assumes a unique set of inspectors parameters θ_V^m associated with each structural category \mathcal{C}_m , which ignores the fact that the same inspector could be responsible for inspections on other structural categories. To overcome this limitation, the new parameter estimation procedure learns the parameters of each inspector I_i based on data from all structural categories, which results in a unified set of inspectors’ parameters θ_V , where $\theta_V = \{\mu_{V_{1:I}}, \sigma_{V_{1:I}}\}$. Accordingly, the model parameters using the new approach are defined as in, $\theta = \{\theta_V, \sigma_W^{(1:5)}, \theta_{\text{BNN}}^{(1:5)}, \theta_0^{(1:5)}\}$. In order to demonstrate the effectiveness of the new parameter estimation procedure, the performance of SSM-BNN(θ_{exist}) and SSM-BNN(θ) are evaluated using the log-likelihood. Table 3 shows the log-likelihoods estimates for the validation and test sets.

Table 3: Comparison of the SSM-BNN(θ) versus the SSM-BNN(θ_{exist}) using five structural categories. The log-likelihood of the SSM-BNN(θ_{exist}) is denoted by $\mathcal{L}_{\text{exist}}$ while the log-likelihood of the joint approach is denoted by \mathcal{L} . **Bold** numbers indicate better performance.

Category	Validation set		Test set	
	$\mathcal{L}_{\text{exist}}$	\mathcal{L}	$\mathcal{L}_{\text{exist}}$	\mathcal{L}
Exterior walls	-1.16E+04	-1.20E+04	-5.47E+03	-5.44E+03
Front walls	-5.79E+03	-5.75E+03	-2.94E+03	-2.92E+03
Slabs	-8.18E+03	-8.13E+03	-3.02E+03	-2.93E+03
Beams	-2.12E+04	-2.10E+04	-1.23E+04	-1.25E+04
Pavement	-6.64E+03	-7.02E+03	-4.69E+03	-3.32E+03

From Table 3, the model SSM-BNN(θ) with the parameters estimated using the new approach performs better on three out of the five structural categories on the validation set and four out of the five structural categories on the test set. These results indicate that the inspectors’ parameters estimated with the new approach generalize better on the independent test sets. Furthermore, the results indicates that training on each structural category independently may induce overfitting (e.g, exterior walls and pavement have better validation log-likelihood and worse test log-likelihood). Training the SSM-BNN

model using data from all the structural categories significantly increases the number of observations per inspector, which contributes to bringing their estimated parameters closer to the true values. This is supported by the model trained using the unified set θ_V achieving better log-likelihoods on most of the validation and test sets (Table 3).

6 Conclusion

This paper proposed a new probabilistic framework for modeling the deterioration of large transportation infrastructure based on visual inspections and structural attributes. The proposed framework relies on a state-space model (SSM) coupled with a Bayesian neural network (BNN) that enables incorporating structural attributes in the deterioration analyses. The SSM-BNN model is trained using a new estimation approach that significantly reduces the number of model parameters. The performance of the SSM-BNN model is verified using synthetic data and validated using real inspection from the network of bridges in Quebec, Canada. The results of the analyses from synthetic data highlights the capacity of the SSM-BNN model in quantifying the relationship between the inferred deterioration speed and the structural attributes. In addition, the SSM-BNN model predictions have a better alignment with the true synthetic speed in comparison to the existing SSM-KR framework. As for the real data, the comparison with the existing SSM-KR shows that the proposed SSM-BNN model achieves approximately 6% improvement in the log-likelihood, while requiring significantly lower training time (3 hours compared to 189 hours). Those advantages have facilitated the implementation of a new parameter estimation procedure that enabled reducing the total number of model parameters while improving the overall generalizability of the model. This is demonstrated through comparisons among the models' log-likelihood estimates on independent validation and test sets. Despite the highlighted advantages in the SSM-BNN model, the use of BNN remains limited to quantifying the initial deterioration speed only. Future work can address this limitation by using frameworks such as LSTM [32], to provide state estimates for timestamps beyond the initial deterioration speed.

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Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

A BNN Configuration and Training

The architecture employed for the BNN model consists of one hidden layer with 128 units, with a ReLU activation function. The weights and biases of the BNN are initialized using He's method [17], and the batch size during the training is 16 instances. The approach used in learning the BNN model parameters is tractable approximate Gaussian inference (TAGI) [10, 7]. The use of TAGI in the context of a fully-connected feedforward neural network (FNN) consists of inputs $\mathbf{s} \in \mathbb{R}^S$ and outputs $\mathbf{r} \in \mathbb{R}^R$ connected by L hidden layers, graphically depicted in Figure 9.

These connections are established through matrix-vector multiplication and addition with the network parameters $\theta_{NN} = \{\mathbf{W}, \mathbf{b}\}$, followed by a non-linear activation function $\varphi(\cdot)$. Specifically, the

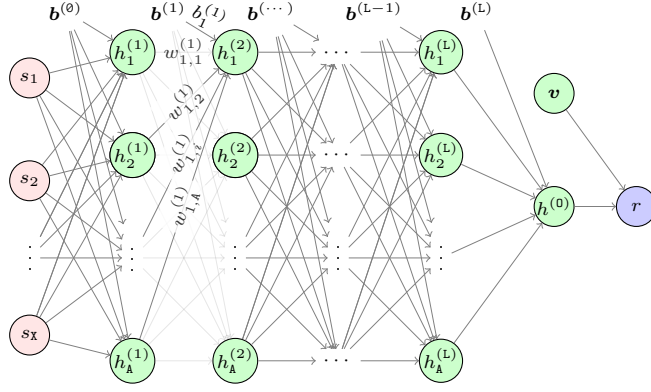


Figure 9: Graphical representation of a feedforward neural network consisting of L hidden layers with A hidden units in each one. The network maps the relation between the inputs $\mathbf{s} = [s_1 \cdots s_x]^\top$ and the unit at the output layer $h^{(0)}$. To simplify visualization, the output is shown here as one-dimensional. The observation r is connected to the output layer unit $h^{(0)}$ and the error term v through the observation model defined in Equation 9. The parameters connecting layer j with layer $j + 1$ consist of a vector of biases $\mathbf{b}^{(j)}$ and a matrix of weights $\mathbf{W}^{(j)}$, such that $w_{m,i}^{(j)} \in \mathbf{W}^{(j)}$ represents the weight connecting the i -th unit in layer j to the m -th unit in layer $j + 1$.

hidden units $\mathbf{h}^{(j)} \in \mathbb{R}^{|j|}$ in layer j are used to obtain the hidden units in the next layer $\mathbf{h}^{(j+1)} \in \mathbb{R}^{|j+1|}$ following

$$\mathbf{h}^{(j+1)} = \mathbf{W}^{(j)} \left(\varphi(\mathbf{h}^{(j)}) \right) + \mathbf{b}^{(j)}, \quad j = 0, 1, \dots, L, \quad (8)$$

where $\mathbf{W}^{(j)} \in \mathbb{R}^{|j+1| \times |j|}$ and $\mathbf{b}^{(j)} \in \mathbb{R}^{|j+1|}$ are the weights and biases in layer j with $|j|$ and $|j + 1|$ indicating the number of units in layers j and $j + 1$, respectively. Note that the activation units in layer $j = 0$ simply correspond to the inputs to the network $\mathbf{s} \equiv \varphi(\mathbf{h}^{(0)})$.

TAGI uses Equation 8 to analytically propagate uncertainty forward by treating the network parameters as independent Gaussian random variables, so that $\boldsymbol{\theta}_{\text{NN}} \sim \mathcal{N}(\boldsymbol{\mu}_{\boldsymbol{\theta}}, \mathbf{I} \cdot \boldsymbol{\sigma}_{\boldsymbol{\theta}}^2)$, as well as making several other assumptions and approximations which are detailed in [10]. The relation between the network's outputs $\mathbf{h}^{(0)}$ and the observed system responses \mathbf{r} is described by the following observation model,

$$\mathbf{r} = \mathbf{h}^{(0)} + \mathbf{v}, \quad \mathbf{v} : \mathbf{V} \sim \mathcal{N}(\mathbf{0}, \mathbf{I} \cdot \boldsymbol{\sigma}_{\mathbf{V}}^2), \quad (9)$$

where \mathbf{v} represents independent zero-mean Gaussian errors. TAGI performs closed-form inference using the equations for the Gaussian conditional and Rauch-Tung-Striebel (RTS) smoother [28], which are detailed in B. Given that TAGI initializes the network parameters with a weakly informed prior, the inference is performed over multiple successive passes over the training set. To prevent overfitting, this iterative learning process is stopped once the likelihood on the validation set stops improving. The use of TAGI in this paper enables the analytical inference of the heteroscedastic observation error variance, the neural network's parameters and hidden units [7].

B Kalman filter and Rauch-Tung-Striebel Smoother

Kalman filter (KF) [19] estimates the deterioration state at each time step t using the prediction and update steps. The prediction step estimates the state \mathbf{x}_t from \mathbf{x}_{t-1} using the transition model from Equation 1, such that,

$$\begin{aligned} \boldsymbol{\mu}_{t|t-1} &= \mathbf{A}\boldsymbol{\mu}_{t-1|t-1}, \\ \boldsymbol{\Sigma}_{t|t-1} &= \mathbf{A}\boldsymbol{\Sigma}_{t-1|t-1}\mathbf{A}^\top + \mathbf{Q}. \end{aligned} \quad (10)$$

Here, $\boldsymbol{\mu}_{t|t-1} \equiv \mathbb{E}[\mathbf{X}_t | y_{1:t-1}]$ and $\boldsymbol{\Sigma}_{t|t-1} \equiv \text{cov}[\mathbf{X}_t | y_{1:t-1}]$ are the expected value and covariance for the state at time t , given all the observations up to time $t - 1$, which are denoted by $y_{1:t-1} \equiv \{y_1, y_2, \dots, y_{t-1}\}$. In the absence of an observation at time t , the prediction step is repeated; otherwise,

if an observation is available, the update step is applied. The update step finds the posterior knowledge of \mathbf{x}_t conditioned on all the observations $y_{1:t}$ using the Gaussian conditional equations.

Once the state estimates are obtained by the KF, they are then refined using the Rauch-Tung-Striebel (RTS) smoother [28], which is given by the following equations:

$$\begin{aligned} f(\mathbf{x}_t | y_{1:T}) &= \mathcal{N}(\mathbf{x}_t; \boldsymbol{\mu}_{t|T}, \boldsymbol{\Sigma}_{t|T}), \\ \boldsymbol{\mu}_{t|T} &= \boldsymbol{\mu}_{t|t} + \mathbf{J}_t (\boldsymbol{\mu}_{t+1|T} - \boldsymbol{\mu}_{t+1|t}), \\ \boldsymbol{\Sigma}_{t|T} &= \boldsymbol{\Sigma}_{t|t} + \mathbf{J}_t (\boldsymbol{\Sigma}_{t+1|T} - \boldsymbol{\Sigma}_{t+1|t}) \mathbf{J}_t^\top, \\ \mathbf{J}_t &= \boldsymbol{\Sigma}_{t|t} \mathbf{A}^\top \boldsymbol{\Sigma}_{t+1|t}^{-1}, \end{aligned} \quad (11)$$

where $\boldsymbol{\mu}_{t|T}$ and $\boldsymbol{\Sigma}_{t|T}$ are the posterior mean and covariance of the smoothed state estimates at time t , given all the observations up to time T , which denotes the time step of the last observation.

C Space transformation

The original observations $\tilde{x} \in [l, u]$ are transformed into an unbounded domain $x \in (-\infty, \infty)$ using the following function,

$$x = o(\tilde{x}) = \begin{cases} \left[\frac{1}{\Gamma(\alpha)} \int_0^{\tilde{x}} t^{\alpha-1} e^{-t} dt \right]^\alpha, & \frac{u+l}{2} < \tilde{x} \leq u, \\ \tilde{x}, & \tilde{x} = \frac{u+l}{2}, \\ -\left[\frac{1}{\Gamma(\alpha)} \int_0^{\tilde{x}} t^{\alpha-1} e^{-t} dt \right]^\alpha, & l \leq \tilde{x} < \frac{u+l}{2}, \end{cases} \quad (12)$$

Here, $\Gamma(\cdot)$ is the gamma function and α is the shape parameter of the gamma distribution. The shape parameter is defined as $\alpha \equiv 2^{-n}$, where n is a positive integer that controls the curvature of the transformation function near the bounds l and u . After completing the analyses, the predictions of the deterioration model $x \in (-\infty, \infty)$ are back-transformed into the original space $\tilde{x} \in [l, u]$ using

$$\tilde{x} = o^{-1}(x) = \begin{cases} \frac{1}{\Gamma(\alpha)} \int_0^{x^{\frac{1}{\alpha}}} t^{\alpha-1} e^{-t} dt, & x > \frac{u+l}{2}, \\ x, & x = \frac{u+l}{2}, \\ -\frac{1}{\Gamma(\alpha)} \int_0^{x^{\frac{1}{\alpha}}} t^{\alpha-1} e^{-t} dt, & x < \frac{u+l}{2}. \end{cases} \quad (13)$$

Further details about the space transformation function are available in the work of Hamida and Goulet [11].

D Learning the Uncertainty of Each Inspector

Each inspector I_i has a unique observation error described by $v_i : V_i \sim \mathcal{N}(\mu_{V_i}, \sigma_{V_i}^2)$. Estimating μ_{V_i} and $\sigma_{V_i}^2$ is done by augmenting the state vector with two additional hidden states defined by,

$$\begin{aligned} v_{b(i)} : V_{b(i)} &\sim \mathcal{N}(v_{b(i)}; \mu_{b(i)}, \sigma_{b(i)}^2), \\ v_{s(i)} : V_{s(i)} &\sim \mathcal{N}(v_{s(i)}; 0, \sigma_{s(i)}^2), \end{aligned} \quad (14)$$

where the term $v_b(I_i)$ is a hidden state corresponding to μ_{V_i} , and the term $v_s(I_i)$ corresponds to $\sigma_{V_i}^2$ [21]. Quantifying these additional hidden states happens concurrently with the inference of the deterioration states in the SSM framework. Specifically, $v_{b(i)}$ and $v_{s(i)}$ are updated whenever the inspection $\tilde{y}_{t,p}^j$ is performed by the inspector I_i . The updates for $v_{b(i)}$ and $v_{s(i)}$ are performed by using the approximate Gaussian variance inference (AGVI) approach for estimating the variance hidden state $v_{s(i)}$, and a recursive framework for estimating the hidden state $v_{b(i)}$ associated with the expected value μ_{V_i} [21].

In principle, AGVI relies on the definition of the variance and the fact that $v_{s(i)}$ has a zero mean [6], such that,

$$\text{var}[V] = \mathbb{E}[V^2] - \mathbb{E}[V]^2 = \sigma_V^2 \equiv \overline{v^2}. \quad (15)$$

Accordingly, the variance can be estimated by quantifying the hidden state $\overline{v^2}$ which can be updated over time based on information from the observations [6]. On the other hand, the hidden state $v_{b(i)}$ is updated recursively from the observations, which is enabled by applying modifications on the transition matrix \mathbf{A} and the observation matrix \mathbf{C} [21]. Further details about the two approaches are provided in the work of Blanche et al. [21].

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